

SM3 9.2 Pyth Trig Proof

1)	$2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$	Given	2)	$\tan^2 \theta = \frac{1 - \cos^2 \theta}{\cos^2 \theta}$	Given
	$2(1 - \sin^2 \theta) - 1 =$	Pyth ID		$\frac{\sin^2 \theta}{\cos^2 \theta} =$	Def of tan
	$2 - 2\sin^2 \theta - 1 =$	Distrib		$\frac{1 - \cos^2 \theta}{\cos^2 \theta} =$	Pyth ID
	$1 - 2\sin^2 \theta =$	Add			
	QED			QED	
3)	$4\sin^2 \theta + 4\cos^2 \theta = 4$	Given	4)	$\cos \theta - \cos^3 \theta = \cos \theta \sin^2 \theta$	Given
	$4(\sin^2 \theta + \cos^2 \theta) =$	Factor		$\cos \theta(1 - \cos^2 \theta) =$	Factor
	$4 =$	Pyth ID		$\cos \theta \sin^2 \theta =$	Pyth ID
	QED			QED	
5)	$\frac{\cos^2 \theta - 1}{\cos \theta} = -\tan \theta \sin \theta$	Given	6)	$\frac{\sec \theta + 1}{\tan \theta} = \frac{\sin \theta}{1 - \cos \theta}$	Given
	$\frac{-(1 - \cos^2 \theta)}{\cos \theta} =$	Factor		$\frac{\frac{1}{\cos \theta} + 1}{\frac{\sin \theta}{\cos \theta}} =$	Def of sec, tan
	$\frac{-\sin^2 \theta}{\cos \theta} =$	Pyth ID		$\left(\frac{1}{\cos \theta} + 1\right) \left(\frac{\cos \theta}{\sin \theta}\right) =$	Division
	$\frac{-\sin \theta \sin \theta}{\cos \theta} =$	Factor		$\left(\frac{1 + \cos \theta}{\cos \theta}\right) \left(\frac{\cos \theta}{\sin \theta}\right) =$	Add
	$-\tan \theta \sin \theta =$	Def of tan		$\frac{1 + \cos \theta}{\sin \theta} =$	Mult
	QED			$\frac{(1 + \cos \theta) \sin \theta}{\sin^2 \theta} =$	Mult
				$\frac{(1 + \cos \theta) \sin \theta}{1 - \cos^2 \theta} =$	Pyth ID
				$\frac{(1 + \cos \theta) \sin \theta}{(1 - \cos \theta)(1 + \cos \theta)} =$	Factor
				$\frac{\sin \theta}{1 - \cos \theta} =$	Divide
	QED			QED	

$$7) \quad \cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta \quad \text{Given}$$

$$(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta) = \quad \text{Factor}$$

$$\cos^2\theta - \sin^2\theta = \quad \text{Pyth ID}$$

QED

$$8) \quad \tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta \quad \text{Given}$$

$$\tan^2\theta(\tan^2\theta + 1) = \quad \text{Factor}$$

$$(\sec^2\theta - 1)(\sec^2\theta) = \quad \text{Pyth ID}$$

$$\sec^4\theta - \sec^2\theta = \quad \text{Distrib}$$

QED

$$9) \quad (1 - \tan\theta)^2 = \sec^2\theta - 2\tan\theta \quad \text{Given}$$

$$1 - 2\tan\theta + \tan^2\theta = \quad \text{Distribute}$$

$$\sec^2\theta - 2\tan\theta = \quad \text{Pyth ID}$$

QED

$$10) \quad (\cos\theta - \sin\theta)^2 = 1 - 2\sin\theta\cos\theta \quad \text{Given}$$

$$\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = \quad \text{Distrib}$$

$$1 - 2\sin\theta\cos\theta = \quad \text{Pyth ID}$$

QED

$$11) \quad \frac{\cos^2\theta}{1 - \sin\theta} = 1 + \sin\theta \quad \text{Given}$$

$$\frac{1 - \sin^2\theta}{1 - \sin\theta} = \quad \text{Pyth ID}$$

$$\frac{(1 - \sin\theta)(1 + \sin\theta)}{1 - \sin\theta} = \quad \text{Factor}$$

$$1 + \sin(\theta) = \quad \text{Divide}$$

QED

$$12) \quad (\sec^2\theta + \csc^2\theta) - (\tan^2\theta + \cot^2\theta) = 2 \quad \text{Given}$$

$$\sec^2\theta - \tan^2\theta + \csc^2\theta - \cot^2\theta = \quad \text{Distrib}$$

$$1 + 1 = \quad \text{Pyth ID}$$

$$2 = \quad \text{Add}$$

QED

$$13) \quad \frac{1}{1 - \cos \theta} + \frac{1}{1 + \cos \theta} = \frac{2}{\sin^2 \theta} \quad \text{Given}$$

$$\frac{1 + \cos \theta}{1 - \cos^2 \theta} + \frac{1 - \cos \theta}{1 - \cos^2 \theta} = \quad \text{Mult}$$

$$\frac{2}{1 - \cos^2 \theta} = \quad \text{Add}$$

$$\frac{2}{\sin^2 \theta} = \quad \text{Pyth ID}$$

QED

$$15) \quad \tan^4 \theta = \tan^2 \theta \sec^2 \theta - \sec^2 \theta + 1 \quad \text{Given}$$

$$\tan^2 \theta \tan^2 \theta = \quad \text{Factor}$$

$$\tan^2 \theta (\sec^2 \theta - 1) = \quad \text{Pyth ID}$$

$$\tan^2 \theta \sec^2 \theta - \tan^2 \theta = \quad \text{Distribute}$$

$$\tan^2 \theta \sec^2 \theta - (\sec^2 \theta - 1) = \quad \text{Pyth ID}$$

$$\tan^2 \theta \sec^2 \theta - \sec^2 \theta + 1 = \quad \text{Distribute}$$

QED

$$14) \quad \frac{\sec^2 \theta \csc \theta}{\sec^2 \theta + \csc^2 \theta} = \sin \theta \quad \text{Given}$$

$$\frac{\frac{1}{\cos^2 \theta} \sin \theta}{\frac{1}{\cos^2 \theta} + \frac{1}{\sin^2 \theta}} = \quad \text{Def of sec, csc}$$

$$\frac{\frac{1}{\cos^2 \theta} \sin \theta}{\frac{\sin^2 \theta}{\cos^2 \theta \sin^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta \sin^2 \theta}} = \quad \text{Mult}$$

$$\frac{\frac{1}{\cos^2 \theta} \sin \theta}{\frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta \sin^2 \theta}} = \quad \text{Add}$$

$$\frac{\frac{1}{\cos^2 \theta} \sin \theta}{\frac{1}{\cos^2 \theta \sin^2 \theta}} = \quad \text{Pyth ID}$$

$$\frac{1}{\cos^2 \theta \sin \theta} \frac{\cos^2 \theta \sin^2 \theta}{1} = \quad \text{Divide}$$

$$\sin \theta = \quad \text{Divide}$$

QED